Question	Scheme	Marks	AOs
1	$\int \frac{3x^4-4}{2x^3}\,dx = \int \frac{3}{2}x - 2x^{-3}\,dx$	M1 A1	1.1b 1.1b
	$= \frac{3}{2}\times\frac{x^2}{2} - 2\times\frac{x^{-2}}{-2} \quad (+c)$	dM1	3.1a
	$= \frac{3}{4}x^2 + \frac{1}{x^2} + c$ o.e	A1	1.1b
		(4)	
		(4 marks)	

Notes:

(i)

M1: Attempts to divide to form a sum of terms. Implied by two terms with one correct index.

$\int \frac{3x^4}{2x^3} - \frac{4}{2x^3}\,dx$ scores this mark.

A1: $\int \frac{3}{2}x - 2x^{-3}\,dx$ o.e such as $\frac{1}{2}\int \left(3x - 4x^{-3}\right)\,dx$. The indices must have been processed on both terms. Condone spurious notation or lack of the integral sign for this mark.

dM1: For the full strategy to integrate the expression. It requires two terms with one correct index. Look for $= ax^p + bx^q$ where $p = 2$ or $q = -2$

A1: Correct answer $\frac{3}{4}x^2 + \frac{1}{x^2} + c$ o.e. such as $\frac{3}{4}x^2 + x^{-2} + c$

Question	Scheme	Marks	AOs
2	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(8x^3 - \frac{3}{2\sqrt{x}} + 5\right) \mathrm{d}x = \frac{8x^4}{4} ... + 5x$	A1	1.1b
	$= ... - 2 \times \frac{3}{2} x^{\frac{1}{2}} + ...$	A1	1.1b
	$= 2x^4 - 3x^{\frac{1}{2}} + 5x + c$	A1	1.1b
		(4)	
		(4 marks)	

Notes

M1: For raising any correct power of x by 1 including $5 \to 5x$ (not for $+ c$) Also allow eg $x^3 \to x^{3+1}$

A1: For 2 correct non-fractional power terms (allow unsimplified coefficients) and may appear on separate lines. The indices must be processed. The $+ c$ does not count as a correct term here. Condone the 1 appearing as a power or denominator such as $\frac{5x^1}{1}$ for this mark.

A1: For the correct fractional power term (allow unsimplified) Allow eg $+ -2 \times 1.5\sqrt{x^1}$.

Also allow fractions within fractions for this mark such as $\frac{\frac{3}{2}}{\frac{1}{2}} x^{\frac{1}{2}}$

A1: All correct and simplified and on one line including $+ c$. Allow $-3\sqrt{x}$ or $-\sqrt{9x}$ for $-3x^{\frac{1}{2}}$. Do not accept $+-3x^{\frac{1}{2}}$ for this mark.

Award once a correct expression is seen and isw but if there is any additional/incorrect notation and no correct expression has been seen on its own, withhold the final mark.

Eg. $\int 2x^4 - 3x^{\frac{1}{2}} + 5x + c \,\mathrm{d}x$ or $2x^4 - 3x^{\frac{1}{2}} + 5x + c = 0$ with no correct expression seen earlier are both A0.

Question	Scheme	Marks	AOs
3(a)	$h = 2.3 - 1.7e^0$	M1	3.4
	Either 0.6 {m} or 60 cm	A1	1.1b
		(2)	
(b)	$\left\{\frac{dh}{dt} =\right\} 0.34e^{-0.2t}$	M1	3.1b
	At $t = 4 \Rightarrow$ Rate of growth is $0.34e^{-0.2\times 4} = 0.15277...\{\text{m / year}\}$	dM1	3.4
	0.153 {m per year} = 15.3 cm {per year} *	A1*	1.1b
		(3)	
(c)	2.3 (m)	B1	2.2a
		(1)	
			(6 marks)

Notes:

(a)

M1: Substitutes $t = 0$ into $h = 2.3 - 1.7e^{-0.2t}$ Implied by e.g., $h = 2.3 - 1.7e^{-0}$ or $h = 0.6$

A1: Allow 0.6, 0.6 m, or 60 cm and isw after a correct height. Allow $\frac{3}{5}$

The M mark may be implied by A1.

(b)

M1: Links rate of change to gradient and differentiates $h = 2.3 - 1.7e^{-0.2t}$ to $k\,e^{-0.2t}$, $k \neq -1.7$

Accept, e.g., $-0.2 \times -1.7e^{-0.2t}$ Must be seen in (b).

dM1: Substitutes $t = 4$ into $k\,e^{-0.2t}$, $k \neq -1.7$ and calculates its value.

A1*: Fully correct. Requires

- sight of $\left\{\frac{dh}{dt} =\right\} 0.34e^{-0.2t}$ o.e., e.g., $\left\{\frac{dh}{dt} =\right\} \frac{17}{50}e^{-0.2t}$ or $\left\{\frac{dh}{dt} =\right\} -0.2 \times -1.7e^{-0.2t}$
- $\left\{\frac{dh}{dt} =\right\}$ awrt 0.153 {metres per year}
- changing to awrt 15.3 cm {per year}.

Note: Substituting $t = 4$ into $h = 2.3 - 1.7e^{-0.2t}$ gives $h = 1.536...$ scores M0dM0A0 unless differentiation and further correct work is seen separately.

(c)

B1: Allow 2.3, 2.3 m, or 230 cm

$2.2\dot{9}$ and 2.2999… which clearly continues are both acceptable, but 2.29999999 is not.

Question	Scheme	Marks	AOs
4	Sets $f'(4)=0 \Rightarrow 16+2a+b=0$	M1	2.1
	Integrates $f'(x)=4x+a\sqrt{x}+b \Rightarrow \{f(x)=\}2x^2+\frac{2}{3}ax^{\frac{3}{2}}+bx\ \{+c\}$	M1 A1ft	1.1b 1.1b
	Deduces that $c=-5$	B1	2.2a
	Full and complete method using the given information $f'(4)=0$ and $f(4)=3$ in order to find values for *a* and *b* Note: $a=-15$ and $b=14$	ddM1	3.1a
	$\{f(x)=\}2x^2-10x^{\frac{3}{2}}+14x-5$	A1	1.1b
		(6)	
		(6 marks)	

Notes:

M1: For the key step in setting $f'(4)=0 \Rightarrow 16+2a+b=0$ to set up an equation in *a* and *b*. Condone slips.

M1: For attempting to integrate $f'(x)$. Award for $x^n \to x^{n+1}$ or $b \to bx$
This may come after finding values for *a* or *b* or both.

A1ft: $\{f(x)=\}2x^2+\frac{2}{3}ax^{\frac{3}{2}}+bx\ \{+c\}$ or, e.g., $\{f(x)=\}2x^2+\frac{2}{3}ax^{\frac{3}{2}}+(-16-2a)x\ \{+c\}$
Allow ft on their *b* in terms of *a* if they substituted in from their $f'(4)=0 \Rightarrow 16+2a+b=0$
Do not ft if they have a value(s) for *a* or *b*
This may be left unsimplified but the indices must be processed.
isw once the mark is awarded. Condone the omission of the + *c*
This accuracy mark requires only the previous M mark to be scored.

B1: Deduces that the constant term in $f(x)$ is -5.
Note that deducing $b=-5$ is B0. It must be the constant in a changed function.

ddM1: For a complete strategy to find values for both *a* and *b*.
Do not be concerned about the logistics of how they solve the simultaneous equations – this may be done on a calculator.
Note: $a=-15$ and $b=14$
This is dependent on **both** previous method marks and so must include use of both

- $f'(4)=0$ (their $16+2a+b=0$ o.e.)
- $f(4)=3$ (their $32+\frac{16}{3}a+4b-5=3$ o.e.)

A1: $\{f(x)=\}2x^2-10x^{\frac{3}{2}}+14x-5$ or exact simplified equivalent, e.g., use of $x\sqrt{x}$ in place of $x^{\frac{3}{2}}$
Apply isw once a correct expression is seen.

Question	Scheme	Marks	AOs
5	$x^{\frac{1}{2}}(2x-5)=...x^{\frac{3}{2}}+...x^{\frac{1}{2}}$ or $\dfrac{x^{\frac{1}{2}}(2x-5)}{3}=\dfrac{...x^{\frac{3}{2}}+...x^{\frac{1}{2}}}{3}$	M1	1.1b
	$\dfrac{2x^{\frac{3}{2}}}{3}-\dfrac{5x^{\frac{1}{2}}}{3}$	A1	1.1b
	$\displaystyle\int \dfrac{2x^{\frac{3}{2}}}{3}-\dfrac{5x^{\frac{1}{2}}}{3}\,dx=...x^{\frac{5}{2}}\pm...x^{\frac{3}{2}}\ (+c)$	dM1	1.1b
	$\dfrac{4}{15}x^{\frac{5}{2}}-\dfrac{10}{9}x^{\frac{3}{2}}+c$	A1	1.1b
		(4)	
			(4 marks)

Notes

M1: Attempts to multiply out the brackets of the numerator and either writes the expression (or just the numerator) as a sum of terms with **indices**. Award for either one correct index of $...x^{\frac{3}{2}}+...x^{\frac{1}{2}}$ which comes from a correct method. Condone appearing as terms on separate lines for this mark. The correct index may be implied later when e.g. $\sqrt{x}\to x^{\frac{3}{2}}$ or $\left(\sqrt{x}\right)^3\to x^{\frac{5}{2}}$ after they have integrated.

The $\frac{1}{3}$ does not need to be considered for this mark.

A1: $\dfrac{2x^{\frac{3}{2}}}{3}-\dfrac{5x^{\frac{1}{2}}}{3}$ or equivalent e.g. $\dfrac{1}{3}\left(2x^{\frac{3}{2}}-5x^{\frac{1}{2}}\right)$. Condone $\dfrac{2x^{\frac{3}{2}}-5x^{\frac{1}{2}}}{3}$ May be implied by further work.

The correct index may be implied later when e.g. $\sqrt{x}\to x^{\frac{3}{2}}$ or $\left(\sqrt{x}\right)^3\to x^{\frac{5}{2}}$ after they have integrated.

Ignore incorrect integration notation around the terms. Ignore any presence or absence of dx.

Be aware that a factor of $\frac{1}{3}$ may be taken outside of the integral so you may need to look at further work to award the first A mark if work on the two terms is done separately or in a list.

May be unsimplified and the two terms may appear in a list which is fine.

Coefficients must be exact.

dM1: Increases the power by one on an x^n term where n is a fraction. The index does not need to be processed.

e.g. $...x^{\frac{3}{2}+1}$ or $...x^{\frac{1}{2}+1}$ It is dependent on the previous method mark so at least one of the terms must have had a correct index.

Note that integrating the numerator and denominator e.g. $\dfrac{2x^{\frac{3}{2}}}{3}-\dfrac{5x^{\frac{1}{2}}}{3}\to\dfrac{...x^{\frac{5}{2}}}{3x}-\dfrac{...x^{\frac{3}{2}}}{3x}$ is dM0.

A1: $\frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c$ and including the constant or simplified exact equivalent such as $\frac{4}{15}\sqrt{x^5}-\frac{10}{9}\sqrt{x^3}+c$ or $\frac{4}{15}x^{2.5}-\frac{10}{9}x^{1.5}+c$ or $\frac{1}{45}\left(12x^{\frac{5}{2}}-50x^{\frac{3}{2}}\right)+c$ or $\frac{x^{\frac{3}{2}}}{45}(12x-50)+c$. Fractions must be in their lowest terms and indices processed.

Do not accept e.g. $0.267x^{\frac{5}{2}}-1.11x^{\frac{3}{2}}+c$ but allow $0.2\dot{6}x^{\frac{5}{2}}-1.\dot{1}x^{\frac{3}{2}}+c$

Isw once a correct answer is seen but withhold this mark if there is spurious notation around their final answer e.g. $\int \frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c\ \mathrm{d}x$ is M1A1dM1A0

Alternative method using integration by parts example

M1: e.g. $\int x^{\frac{1}{2}}(2x-5)\ \mathrm{d}x = ...x^{\frac{3}{2}}(2x-5)-\int ...x^{\frac{3}{2}}\ \mathrm{d}x$ (applies integration by parts correctly to typically achieve this form – the $(2x-5)$ may also be split up as well – send to review if unsure how to mark)

This may also be done the other way round e.g. $\int x^{\frac{1}{2}}(2x-5)\ \mathrm{d}x = ...x^{\frac{1}{2}}(x^2-5x)-\int ...x^{\frac{3}{2}} \pm ...x^{\frac{1}{2}}\ \mathrm{d}x$

The $\frac{1}{3}$ does not need to be considered for this mark.

A1: A correct intermediate stage applying integration by parts with correct coefficients.

e.g. $\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\ \mathrm{d}x = \frac{2}{3}x^{\frac{3}{2}}\left(\frac{2x-5}{3}\right)-\int \frac{4}{9}x^{\frac{3}{2}}\ \mathrm{d}x$ (or unsimplified equivalent).

Coefficients must be exact. (See main scheme notes above) The other way round this could appear as

e.g. $\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\ \mathrm{d}x = \frac{1}{3}x^{\frac{1}{2}}(x^2-5x)-\frac{1}{6}\int x^{\frac{3}{2}}-5x^{\frac{1}{2}}\ \mathrm{d}x$. Condone a missing d$x$. May be implied.

dM1: Increases the power by one on an x^n term where n is a fraction e.g. $\int ...x^{\frac{3}{2}}\ \mathrm{d}x \to ...x^{\frac{5}{2}}$ The index does not need to be processed. It is dependent on the previous method mark.

A1: $\frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c$ or exact simplified equivalent. (See main scheme notes above)

Alternative method using the substitution method

M1: e.g. let $u=x^{\frac{1}{2}} \Rightarrow \int ...u^4+...u^2\ \mathrm{d}u$ (uses a substitution to express the integral in terms of another variable. Allow slips with the coefficients, but the indices should be correct for their substitution)

The $\frac{1}{3}$ does not need to be considered for this mark.

A1: e.g. $\int \frac{4u^4}{3}-\frac{10u^2}{3}\ \mathrm{d}u$ or unsimplified equivalent. Coefficients must be exact. See main scheme notes above). May be implied by further work. Condone a missing dx.

dM1: $\int ...u^4+...u^2\ \mathrm{d}u \to ...u^5+...u^3$ (increases the power by one on at least one of their indices – does not need to be processed. It is dependent on the previous method mark.

A1: $\frac{4}{15}x^{\frac{5}{2}}-\frac{10}{9}x^{\frac{3}{2}}+c$ or exact simplified equivalent. (See main scheme notes above)

There may be alternative substitutions, but the same marking principles apply.

Question	Scheme	Marks	AOs
6(a)	$2(3)^3-9(3)^2+5(3)+k=0\Rightarrow k=...$	M1	1.1b
	$54-81+15+k=0\Rightarrow k=12*$ or $-12+k=0\Rightarrow k=12*$	A1*	1.1b
		(2)	
	(a) Alternative by verification:		
	$2(3)^3-9(3)^2+5(3)+12=0$	M1	1.1b
	$54-81+15+12=0$ Hence $k=12$ *	A1*	1.1b
		(2)	
(b)	$\int(2x^3-9x^2+5x+12)\,dx ...x^4 \pm ...x^3 \pm ...x^2 \pm ...x \pm ...$	M1	3.1a
	$\frac{1}{2}(3)^4-3(3)^3+\frac{5}{2}(3)^2+12(3)+c=-10\Rightarrow c=...$	dM1	1.1b
	$(0,\ -28)$	A1	2.2a
		(3)	
			(5 marks)

Notes

(a) **Mark (a) and (b) together**

M1: Substitutes $x = 3$ completely into the given derivative, sets = 0 and solves for k.

e.g., $2(3)^3-9(3)^2+5(3)+k=0\Rightarrow k=...$

May be implied by e.g. $54-81+15+k=0\Rightarrow k=...$ with at least 2 correctly evaluated powers.

A1*: Obtains $k = 12$ with no errors seen and sufficient working shown. As a minimum you would need to see e.g., $2(3)^3-9(3)^2+5(3)+k=0\Rightarrow -12+k=0\Rightarrow k=12$ **or** $54-81+15+k=0\Rightarrow k=12$ **or** $2\times27-9\times9+5(3)+k=0\Rightarrow k=12$

But $2(3)^3-9(3)^2+5(3)+k=0\Rightarrow k=12$ scores M1A0 for lack of working

Note that some are just writing the expression for $\frac{dy}{dx}$, then write "sub in $x = 3$" but don't actually show 3 substituted in and then go on to write $-12+k=0$ leading to $k=12$ scores M0A0*.

Alternative:

M1: Substitutes $x = 3$ and $k = 12$ into the given derivative and attempts to evaluate

A1*: Correct work to obtain an answer of 0 with a (minimal) conclusion e.g., tick, hence proven etc.

As a minimum you would need to see e.g., $2(3)^3-9(3)^2+5(3)+12=54-81+15+12=0$ ✓

(b)

M1: Attempts to integrate. Evidence can be taken for integrating to obtain at least 2 from:

$2x^3\to...x^4$ or $-9x^2\to...x^3$ or $5x\to...x^2$ or $12\to...x$ where … are constants

dM1: Substitutes $x = 3$ into their integrated expression that includes a constant of integration, sets this equal to ±10 and proceeds to find their constant. **Depends on the previous mark.**

If the substitution is not shown this mark may be implied by their value for c or by their equation e.g., $18+c=\pm10$

A1: $(0,\ -28)$ Condone -28 or $y = -28$ but not just $c = -28$. There must be no other values or points.

Condone $(-28,\ 0)$ following $y = -28$

<u>Beware of circular arguments which avoid doing part (a) e.g.</u>

Integration is used on the given derivative to give y in terms of x, k and c

$(3, -10)$ is substituted to give $3k + c = 8$

Part (b) is then done first using $k = 12$ to find $c = -28$

This is then substituted into $3k + c = 8$ to give $k = 12$
This scores (a) M0A0 (b) M1dM1A1 (if –28 is identified as the intercept)

Alternative for part (a) using algebraic division:

$$\begin{array}{r} 2x^2 - 3x \quad - 4 \\ x-3 \overline{\big) 2x^3 - 9x^2 + 5x + k} \\ \underline{2x^3 - 6x^2} \qquad\qquad \\ -3x^2 + 5x \qquad \\ \underline{-3x^2 + 9x} \qquad \\ -4x + k \\ \underline{-4x + 12} \\ k - 12 \text{ (or 0)} \end{array}$$

leading to $k - 12 = 0$ and then $k = 12$.

M1: Attempts to divide the given cubic by $(x-3)$ and proceeds as far as a remainder set = 0. Requires at least $2x^2 \pm 3x$.

A1*: Obtains $k = 12$ with no errors seen and sufficient working. Their algebraic division needs to be correct but allow them to have either $k - 12$ or 0 as their "remainder". If their remainder is given in their working as 0 they may proceed directly to $k = 12$.